DOCUMENT RESUME

ED 241 539 TM 820 764

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TITLE Construct Validity of the GRE Aptitude Test across Populations--An Empirical Confirmatory Study.
INSTITUTION Educational Testing Service, Princeton, N.J.; Graduate Record Examinations Board, Princeton, N.J.
REPORT NO ETS-RR-81-57; GREB-78-1P
PUB DATE Jun 82
NOTE 62p.
AVAILABLE FROM Educational Testing Service, Publications Order Services, Dept. I-101, Princeton, NJ 08541
PUB TYPE Reports - Research/Technical (143)

EDRS PRICE MF01/PC03 Plus Postage.
DESCRIPTORS Black Students; *College Entrance Examinations; Factor Structure; Graduate Study; Higher Education; Item Analysis; Psychometrics; *Racial Differences; Scores; Statistical Analysis; *Test Bias; Test Items; *Test Validity; White Students
IDENTIFIERS Confirmatory Factor Analysis; *Graduate Record Examinations

ABSTRACT

The study evaluated the invariance of the construct validity and thus the interpretation of Graduate Record Examinations (GRE) Aptitude Test scores. A systematic procedure for investigation of test bias from a construct validity frame of reference was developed and applied. Invariant construct validity was defined as similar patterns of loadings and equal units of measurement across four populations, as well as equal test score precision as defined by the standard error of measurement. The advantage of investigating psychometric bias at the item type level was to restrict use of a biased item type. The study also evaluated the factor structure of the subtests--Verbal, Quantitative, and Analytical. The subpopulations in the study were black and white males, and black and white females. The results indicated the item type factors appeared to be measuring the same things in the same units with the same precision. There was no significant evidence of psychometric bias. The factorial contributions of the Analytical item types were investigated. Analysis results indicated that the three Analytical item types appeared to be varying functions of reading comprehension and quantitative ability. (DWH)

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GRE

GRADUATE RECORD EXAMINATIONS

CONSTRUCT VALIDITY OF THE GRE APTITUDE TEST ACROSS POPULATIONS--AN EMPIRICAL CONFIRMATORY STUDY

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GRE Board Professional Report GREB No. 78-1P
ETS Research Report 81-57

June 1982

This report presents the findings of a research project funded by and carried out under the auspices of the Graduate Record Examinations Board.

EDUCATIONAL TESTING SERVICE, PRINCETON, NJ

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CONSTRUCT VALIDITY OF THE GRE APTITUDE TEST ACROSS POPULATIONS--

AN EMPIRICAL CONFIRMATORY STUDY

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GRE Board Professional Report GREB No. 78-1P

June 1982

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Abstract

The purpose of this study was to: (1) evaluate the invariance of the construct validity and thus the interpretation of GRE Aptitude Test scores across four populations, and (2) develop and apply a systematic procedure for investigating the possibility of test bias from a construct validity frame of reference. The notion of invariant construct validity was defined as: (1) similar patterns of loadings across populations; (2) equal units of measurement across populations; and (3) equal test score precision as defined by the standard error of measurement. If any one of the above criteria differs across populations, then one has to consider seriously the possibility of psychometric bias, as defined in this paper. The advantage of investigating psychometric bias at the item type level (even though the total score may not be biased) is that this may provide an "early warning" with respect to any future plans to increase the number of items of any particular type. A secondary purpose of this study was to evaluate the factor structure of the three sections (verbal, quantitative, and analytical) on which the subscores are derived. Assuming that the invariant construct validity model based on item types is tenable, a hypothesized three factor "macro" model based on the three sections could be applied to the population invariant variance-covariance matrix.

It should be noted that the term "psychometric bias" as defined here does not require external criteria information for the analysis. The internal procedure used here is suggested as only a first step in a broader process of an integrated validation procedure that should include not only internal checks on the population invariance of the underlying constructs but also checks on the population invariance of their relationship with external criteria. Although this is only a first step, it is a necessary step since any interpretation of relationships with external criteria becomes academic unless one can first show that the tests measure what they purport to measure with similar meaning and accuracy for all populations of interest.

The four subpopulations were 1,122 White males, 1,471 White females, 284 Black males, and 626 Black females.

The analysis indicated that a factor structure defined by the 10 item types showed relatively invariant psychometric characteristics across the four subpopulations. That is, the item-type factors appear to be measuring the same things in the same units with the same precision. These results do not provide any significant evidence of psychometric bias in the test.

Confirmatory analysis of a higher-order factor model defined by an a priori model based on three- and four-factor solutions was attempted to investigate the factorial contributions of the analytical item types. Results of this analysis indicated that the three analytical item types appear to be varying functions of reading comprehension and quantitative ability. The analysis of explanations item type was the more complex factorially and included a vocabulary component as well as reading and quantitative components. Of the remaining two analytic item types, logical diagrams had the comparatively larger unique variance component. Analytical reasoning appeared to share most of its variance with the reading comprehension and quantitative factors.

Construct Validity of the GRE Aptitude Test Across Populations--An Empirical Confirmatory Study

D. A. Rock, C. Werts, and J. Grandy

Introduction

Construct validation is the basic prerequisite to proper interpretation of a test score. Any time an educator asks "But what does the instrument really measure?" information on construct validity is being requested (e.g., see Cronbach, 1971). Construct validation is the process of marshalling evidence of relationships with other variables to support the inference that an observed test score has a particular meaning; for example, that it is a valid measure of developed verbal or mathematical ability. Implicit in this definition is the presence of an a priori theory or model that in turn generates predictions about expected correlational patterns among measures of the construct of interest as well as with measures of other relevant constructs.

The presence of empirical findings that are consistent with the a priori model furnishes support for the construct validity of the measuring instrument. Empirical findings that are at variance with the a priori model either cast doubt on interpretation of the test score or at best limit its interpretation (Campbell & Fiske, 1959).

Operationally, this study attempts to accomplish two goals. First, it investigates the stability of item type factor interrelationships as well as thier psychometric characteristics across

Black male, Black female, White male, and White female populations. Secondly, it examines the convergent and discriminant validity of the verbal, quantitative, and analytical ability sections of the GRE Aptitude Test. The term convergent validity simply means that the item types that are assumed to be measures of a hypothetical construct such as analytical ability should demonstrate proportionately higher interrelationships among themselves than with measures of other constructs such as verbal or quantitative ability. The term discriminant validity suggests that hypothetical constructs such as verbal, quantitative, and analytical ability are more usefully interpreted if they can be shown empirically to be measuring different things.

Recent procedures in maximum likelihood confirmatory factor analysis (SÖrbom, 1974) allow researchers to: (1) test for "goodness of fit" an a priori factor pattern model based on item types; (2) estimate and test equality of units of measurement for equivalent item-type sections; (3) estimate and test the reliability or accuracy with which each of the item-type factors are measured; and (4) test the invariance of the item-type factors across populations. That is, does the test measure the same things in the same units with equal precision for all subpopulations? If the data do not confirm that the test is measuring the same things in the same units across subpopulations, then the test score interpretations must be called into question.

The GRE verbal, quantitative, and analytical ability sections can

be subdivided into 10 subsections based on item-type classifications. If it can be shown at this relatively micro level (i.e., the item-type level) that the 10 item-type factors are measuring the same things with the same accuracy across all populations, then we can use the maximum likelihood (MLH) estimate of the population invariant variance-covariance matrix resulting from the best fitting factor model to investigate the relationships between item types and the developed abilities they purport to measure. That is, using the MLH estimate of the population invariant variance-covariance matrix, one can confirm or disconfirm an a priori model in which the four verbal item types define a verbal factor, the three quantitative item types define a quantitative factor, and an analytical factor is defined by its three respective item types. Such an analysis will confirm the usefulness of maintaining these separate scores as well as provide information on the psychometric contribution of the respective item types to their underlying factor or construct.

Testing the Invariance of Psychometric Characteristics Across Populations

The first step is to examine the comparability of the pattern of loadings across populations. Assuming that one finds empirical evidence for the similarity of the pattern of factor loadings on the hypothesized item factors in each population, then one can ask whether the scale units for the reading factor, analogy factor, etc. are the same across populations. Being tested here is whether the corresponding factor loadings are the same across populations when the factors are given the observed units of one of thier indicator variables. That is, if we

hypothesize that a given factor, e. g., the reading comprehension factor, can be defined by two split-halved scores from the reading comprehension section, and the factor is given the raw score units of the odd-item half, then if the model is correct, the factor loadings for the even and odd item subtest scores should be equivalent both within and across populations. The important point here, however, is not so much whether the two reading comprehension split halves are tau equivalent within each population (i.e., have equivalent odd and even factor loadings in raw score units) but whether they maintain their proportionality ratio across the populations.

If the scale units are found to be different in one or more populations, one must conclude that the interpretation of the observed scores may not be equivalent across populations. Such a situation is the internal or psychometric counterpart of the "test bias" definition that argues that a test is biased against one group or another if the slopes of the regressions of an external criterion on the test are not the same (e.g., see Cleary, 1968). However, in this case we are comparing the slope of the observed scores on the true scores across groups or populations. As Jöreskog (1971) points out, if the variables that define each factor can be shown to be at least congeneric (i.e., measures of the same thing as indicated by similar patterns of salient loadings), then the maximum likelihood estimates of the raw score factor loadings are the regressions of the observed scores on their "true" scores. If the corresponding raw score factor loadings are

equal, then we would expect that the true score difference corresponding to a particular observed score difference would be uniform across populations. The reader should note here that we are referring to the maximum likelihood "raw score" factor loading estimated from the variance-covariance matrix and not the traditional standardized loadings derived from least squares solutions applied to a correlation matrix. Such standardized solutions can neither estimate nor test the equivalence of measurement units across populations.

In addition to gathering empirical evidence that a test is measuring the same things in the same units, one should also demonstrate that the test is measuring with the same precision across all populations. That is, a third, albeit less serious, indicator of possible psychometric bias is the finding of the nonequivalence across populations of the precision with which each factor or construct is measured. Specifically, are the standard errors of measurement of the factors underlying the test the same across all populations? Tests of the equivalence of the standard errors of measurement are only meaningful, however, if we have first shown that we are measuring the same things in the same scale units. The standard error of measurement is preferable to the traditional reliability estimates as an indicator of a test score's precision since it is more likely to be invariant across populations that differ with respect to the amount of variability in the trait being measured. When one is comparing the precision of test scores across populations characterized by differing variability with respect to the trait of

interest, the traditional reliability indices confound population heterogeneity with measurement error (see, for example, Wiley, 1973).

The question arises, Why investigate the invariance of the psychometric characteristics of the GRE Aptitude Test through the use of item types rather than through the use of other categories such as content areas? There are a number of practical and theoretical reasons for the choice of item types rather than content areas or processes. First, the test specifications with respect to item types are relatively stable, both across form and time of administration. Second, the three subscores presently used are defined by item types. Third, item types can be thought of as different methods of measuring their respective constructs, and previous research (Campbell & Fiske, 1959; Rock & Werts, 1979) suggests that method factors are present and are significant sources of variance. Fourth, since this is a confirmatory analysis whose goal is to investigate the invariance of the psychometric properties of the GRE Aptitude Test, an objective means for conveniently classifying items to form an a priori factor model is necessary.

Purpose

The primary purpose of this study is to evaluate the invariance of the construct validity of the GRE Aptitude Test and thus interpretation of the test scores across four populations. The subpopulations we will be concerned with here are White males, White females, Black males, and Black females. The notion of invariant construct validity is defined as (1) similar patterns of loadings across populations, (2) equal units of measurement across populations, and (3) equal test score precision as defined by the standard error of measurement. If any one of these criteria differs across populations, then one has to consider seriously the possibility of psychometric bias, as defined in this paper. The advantage of investigating psychometric bias at the item-type level (even though the total score may not be biased) is that this may provide helpful information with respect to any test development decisions concerning item-type representation in the total test specifications. A secondary purpose of this study is to evaluate the factor structure of the three sections (verbal, quantitative, and analytical ability) on which separate scores are derived. Assuming that the invariant construct validity model based on item types is tenable, this hypothesized three-factor "macro" model will be carried out on the maximum likelihood estimate of the population invarient variance-covariance matrix.

Sample

Scores were gathered for a total of 3,503 social science majors who were also American citizens and were taking the GRE Aptitude Test for

the first time. These individuals were part of the September 1978 test administration. The total sample was further divided into four subpopulations: 1,122 White males, 1,471 White females, 284 Black males, and 626 Black females. These four subpopulations were used in the subsequent comparisons of the factor models. The matching on major field, etc. was carried out in an effort to minimize the possibility of confounding other background factors with the effects of racial and sex group memberships.

Method

Sörbom and Jöreskog's (1976) program for confirmatory factor analysis across populations, COFAMM, was used to test the various explicit assumptions about the invariance of the construct validity of the GRE Aptitude Test across populations.

COFAMM assumes that a factor analysis model holds in each of the g populations under study. If $\underset{\sim g}{x}$ is defined as the vector of the p observed measures in group g, then $\underset{\sim g}{x}$ can be accounted for by k common factors ($\underset{\sim g}{f}$) and p unique factors ($\underset{\sim g}{z}$). The model in each population is:

$$\underset{\sim g}{x} = \underset{\sim g}{\nu} + \underset{\sim g}{\Lambda}\underset{\sim g}{f} + \underset{\sim g}{z} \qquad (1)$$

where $\underset{\sim g}{\nu}$ is a p x 1 vector of location parameters and $\underset{\sim g}{\Lambda}$ a p x k matrix of factor loadings. It is assumed that $\underset{\sim g}{z}$ and $\underset{\sim g}{f}$ are uncorrelated, the expectation of $\underset{\sim g}{z} = \underset{\sim}{0}$, and the expectation of $\underset{\sim g}{f} = \underset{\sim g}{\theta}$, where $\underset{\sim g}{\theta}$ is a k x 1 parameter vector.

Given these assumptions, the mean vector $\mu_{~g}$ of the $x_{~g}$ is

$$\mu_{~g} = \nu_{~g} + \Lambda_{~g} \theta_{~g} \tag{2}$$

and the expected variance-covariance matrix $\Sigma_{~g}$ of $x_{~g}$ is

$$\Sigma_{~g} = \Lambda_{~g} \phi_{~g} \Lambda_{~g}^{\prime} + \Psi_{~g} \tag{3}$$

where $\phi_{~g}$ is the variance-covariance matrix of $f_{~g}$ and $\Psi_{~g}$ is the variance-covariance matrix of $z_{~g}$. When the factor model does not fit the data perfectly, the observed variance-covariance matrices $S_{~g}$ and observed means will differ from the maximum likelihood estimates of $\Sigma_{~g}$ and $\mu_{~g}$. The program yields a chi-square statistic that is a measure of these differences; that is, of how well the model fits the data.

The four matrices, $\theta_{~g}$, $\Lambda_{~g}$, $\phi_{~g}$, and $\Psi_{~g}$, are called the pattern matrices. The elements of these matrices are the model parameters, which are of three kinds: (a) fixed parameters, which have been assigned given values, like 0 or 1; (b) constrained parameters, which are unknown but equal to one or more other parameters; and (c) free parameters, which are unknown and not constrained to be equal to any other parameter. A parameter may be constrained to be equal to other parameters in the same and/or different pattern matrices in the same and/or different groups.

An important feature of a confirmatory analysis is that the parameters of the model may be uniquely estimated, i.e., the model is identified. A solution is unique if all linear transformations of the factor that leave the fixed parameters unchanged also leave

the free parameters unchanged. It is difficult in general to give useful conditions that are sufficient for identification. However, at one point in the program the information matrix for the unknown parameters is computed. If this matrix is positive definite, it is almost certain that the model is identified. If this matrix is not positive definite, the program prints a message to this effect, specifying which parameter is probably not identified.

In all succeeding tests of these data the models are over-identified, yielding not only unique solutions but sufficient degrees of freedom for a statistical test of "goodness of fit." If the model is identified, as in these examples, standard errors for all the unknown parameter estimates are also provided by the program.

Results and Discussion

Tests of the 10 Factor Item-Type Model

The factor pattern of the GRE Aptitude Test is a special case of equation (1) with the number of variables equal to 20 and the number of hypothesized factors equal to 10. The factor pattern is defined by 10 item-type factors, each of which is identified by two observed variables. The two observed indicators of each factor are scores on odd-even halves for each item type yielding a total of 20 scores--two scores defining each item-type factor. In terms of equation (1) the 10 item-type factors generate the constrained loading pattern shown in Figure 1.

Figure 1

Hypothesized Factor Loading Pattern

Sentence Completion Odd	x_1		v_1		1	0	0	0	0	0	0	0	0	0	f_1
Sentence Completion Even	x_2		v_2		λ_{21}	0	0	0	0	0	0	0	0	0	f_2
Analogies Odd	x_3		v_3		0	1	0	0	0	0	0	0	0	0	f_3
Analogies Even	x_4		v_4		0	λ_{42}	0	0	0	0	0	0	0	0	f_4
Antonyms Odd	x_5		v_5		0	0	1	0	0	0	0	0	0	0	f_5
Antonyms Even	x_6		v_6		0	0	λ_{63}	0	0	0	0	0	0	0	f_6
Reading Odd	x_7		v_7		0	0	0	1	0	0	0	0	0	0	f_7
Reading Even	x_8		v_8		0	0	0	λ_{84}	0	0	0	0	0	0	f_8
Quantitative Comparison Odd	x_9		v_9		0	0	0	0	1	0	0	0	0	0	f_9
Quantitative Comparison Even	x_{10}	=	v_{10}	+	0	0	0	0	λ_{105}	0	0	0	0	0	f_{10}
Regular math Odd	x_{11}		v_{11}		0	0	0	0	0	1	0	0	0	0	
Regular math Even	x_{12}		v_{12}		0	0	0	0	0	λ_{126}	0	0	0	0	
Data Interpretation Odd	x_{13}		v_{13}		0	0	0	0	0	0	1	0	0	0	
Data Interpretation Even	x_{14}		v_{14}		0	0	0	0	0	0	$\lambda_{14,}$	0	0	0	
Analysis of Explanations Odd	x_{15}		v_{15}		0	0	0	0	0	0	0	1	0	0	
Analysis of Explanations Even	x_{16}		v_{16}		0	0	0	0	0	0	0	λ_{168}	0	0	
Logical Diagrams Odd	x_{17}		v_{17}		0	0	0	0	0	0	0	0	1	0	
Logical Diagrams Even	x_{18}		v_{18}		0	0	0	0	0	0	0	0	λ_{189}	0	
Analytical Reasoning Odd	x_{19}		v_{19}		0	0	0	0	0	0	0	0	0	1	
Analytical Reasoning Even	x_{20}		v_{20}		0	0	0	0	0	0	0	0	0	λ_{2010}	

$$\underset{-g}{x} = \underset{-g}{v} + \underset{-g}{\Lambda}\, \underset{g}{f} + \underset{-g}{z}$$

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One factor loading in each column is fixed at unity in order to scale each factor arbitrarily in terms of the observed units of its lead indicator. Thus, each factor is assumed to be determined by its split-halved odd and even item subtest scores. Consequently, we are testing a "pure" simple structure derived from the original test specifications that dictate 10 "pure" item-type factors across all populations. We have put all our constraints in Λ (i.e., the 180 loadings constrained to be zero plus the 10 loadings constrained to unity) and left the 10 x 10 factor variance-covariance matrix (ϕ) and the 20 x 20 diagonal matrix of errors or uniqueness to be estimated.

Since there are 210 unique observed elements in any given sample variance-covariance matrix, S_g ($g = 1, 2, \ldots, 4$), and we are only estimating 55 unknown factor variance-covariances in ϕ, 20 unknown uniquenesses in ψ, and 10 unknown factor loadings in Λ, we have 125 degrees of freedom (210-85) for testing "goodness of fit" within any given population. However, since we are testing the invariance of this factor pattern across four populations, we have 4 x 125 = 500 total degrees of freedom for our test. We shall add the additional constraint that the intercepts, v_g ($g = 1, 2, \ldots, 4$), be equal across populations. This is of course a special case of equation (1) that will become more meaningful when we constrain the factor loadings to be equal across populations. At that point, we shall discuss this constraint in more detail.

If the GRE 10 factor model is consistent with the data, then the difference between the observed variance-covariance matrix (S_g) and the constrained populations variance-covariance matrix (Σ_g) is essentially a null matrix for all groups. The interpretation would be that there are 10 item-type factors present in all populations and the respective indicators of each factor are at least congeneric (i.e., odd and even subtest scores are measuring the same things in all populations). Since we did not constrain corresponding unknown nonzero factor loadings to be equal across populations, we cannot yet make the stronger statement that the subtest scores are not only measuring the same thing but also have the same units of measurement.

The statistical test of the hypothesis of equivalent constrained factor patterns and equal intercepts across populations yielded a χ^2 of 375 with 530 degrees of freedom ($p = .999$).

A more appropriate measure of "goodness of fit" in such large samples is the matrix of differences between corresponding elements in each population's observed variance-covariance matrix S_g and the reproduced variance-covariance matrix (Σ_g) conditional on the constrained factor model. Unfortunately, it is not easy to interpret these discrepancies in the case of the variance-covariance matrices. Therefore, the within-population observed and reproduced variance-covariance matrices were rescaled as correlation matrices. The root mean square (RMS) of these standardized residuals may be interpreted as you would interpret the residuals when fitting a

factor model to the observed correlation matrix.

The RMS residuals within each population for the equivalent factor pattern model with the corresponding intercepts constrained to be equal across populations are presented in Table I.

Table I

Root Mean Square Residuals by Population for the Factor Model that Assumes the Same Pattern of Factor Loadings and Equal Intercepts Across Populations

	White Male	White Female	Black Male	Black Female
Root Mean Square Residuals for Means	.0805	.0545	.0325	.0256
Root Mean Square Residuals When the Variance-Covariance Matrix is Rescaled within Populations to be a Correlation Matrix	.0180	.0189	.0313	.0258

Average of All Root Mean Square Residuals = .0359

The RMS residuals for the means shown in Table I are based on the discrepancy between the observed means and the predicted means conditional on the constrained pattern of factor loadings and the further imposition of equality of intercepts. As pointed out earlier, this latter restriction becomes relevant in the following section on equal scale units and a more detailed explanation is presented there.

Clearly the constrained 10-factor solution fits quite well for all four populations. The near zero residuals confirm that similar factor patterns of zero and nonzero loadings are present in all four populations.

Equal Scaling Units Across Populations

If, in addition to observing the same pattern of factor loadings in all populations, as was done in the previous step, the corresponding factor loadings themselves are also constrained to be equal across populations, then we can test whether the factors have the same scale units across all populations. The hypothesis being tested may be stated as follows: H_0: $\Lambda_1 = \Lambda_2, \ldots, \Lambda_4$ conditional on a 10-factor solution with equal intercepts across populations against the alternative H_1: $\Lambda_1 \neq \Lambda_2 \neq, \ldots, \Lambda_4$ given a 10-factor solution and equal intercepts. Since the factor loadings are maximum likelihood (MLH) estimates of the regressions of the observed scores on the true scores, the constraint of equality across populations is equivalent to a test of equality of scaling units (when the measures are congeneric, i.e., have the same factor loading patterns and also have the same intercepts). However, the individual odd-even halves within populations are not assumed to have equal scales, i.e., equal loadings or intercepts. In this case 30 additional degrees of freedom are gained over the previous test of 10 factoredness since a total of 30 more constraints have been added. This more restricted model led to an increment in χ^2 over the previous test (i.e., the test of

similar factor patterns and the intercepts with no equality constraints across populations on the nonzero factor loadings) of 59 with 30 degrees of freedom ($p \leq .001$). Thus this hypothesis would be rejected on purely statistical grounds. However, the large sample size almost guarantees that very small deviations from the hypothesis would lead to statistical significance. As with the previous test, since the large sample size makes the usual interpretation of statistical tests less meaningful, we will opt for the root mean square residuals as the primary measure of "goodness of fit." Table II below shows the root mean square residuals by population for this more constrained model. Here it is clear that the residuals are quite small even though they produce a statistically significant departure from the model. It seems reasonable to conclude that the model provides a reasonably good fit to the data across the four subpopulations--that the item types measure essentially the same things in essentially equal units across all populations.

Table II

Root Mean Square Residuals by Population for the Factor Model that Assumes Equal Factor Patterns and Equal Intercepts Across Populations

	White Male	White Female	Black Male	Black Female
Root Mean Square Residuals for Means	.0523	.0626	.0363	.0350
Root Mean Square Residuals When the Variance-Covariance Matrix is Rescaled within Populations to be a Correlation Matrix	.0232	.0225	.0340	.0315

Average of All Root Mean Square Residuals = .0372

Comparison of the root mean square residuals in this model with those of the previous less restrictive model indicates that the "goodness of fit" suffered little from the additional imposition of equal scaling units across populations. That is, there is still no practical difference between the average residual and zero. The results suggest that the item types do define the same factors in the same units across all populations. At this point it might be helpful to pictorially present the relationships between the concept of equal scale units for each item and the imposed constraints on equality of factor patterns and intercepts.

The question here is: Given equality constraints on the factor pattern, and thus the units of measurement, are the factor means (Θ_g) consistent with the observed means?

This is a special case of equation (2) that formally defined is:

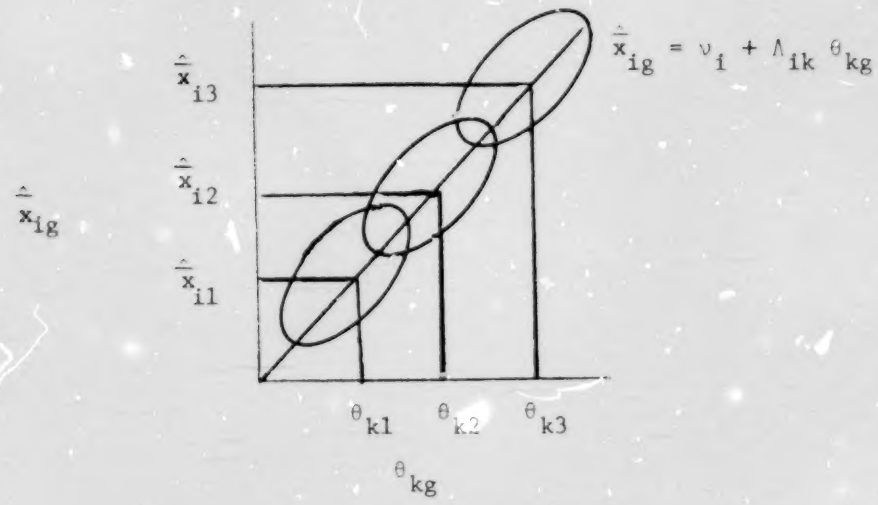
$$\underset{\sim}{u}_g = \underset{\sim}{\nu} + \underset{\sim}{\Lambda}\,\underset{\sim}{\Theta}_g \qquad (4)$$

where $\underset{\sim}{u}_g$ is the vector of observed means, $\underset{\sim}{\nu}$ is the vector of intercepts constrained to be equal across all groups, $\underset{\sim}{\Theta}g$ is the vector of factor mean scores free to vary across populations, and $\underset{\sim}{\Lambda}$ the matrix of maximum likelihood factor loadings equivalent to the regressions of observed scores on true scores, given equivalent factor loading patterns. This relationship can be best expressed by Figure 2.

Figure 2

The Regression of the Observed Means on

Means for Three Different Hypothetical Populations



Where $\bar{x}_{ig}$ = the observed mean on the ith variate in the gth population

ν_i = the intercept constrained to be equal across the populations

Λ_{ik} = the regression of the observed scores on the "true" scores (factor loading) constrained to be equal across populations

θ_{kg} = mean "true" score for the kth factor in the gth population

Under the present model there are, of course, four populations, all of which are assumed to be lying along the same regression line and differing only in their factor means (θ_{kg}). The populations may have different ability levels as reflected by different true score means, but the intercept and the multiplicative or scaling parameter, Λ, must be the same or one must question whether or not that particular item type is measuring in the same scale units in all populations. When the scaling parameters are different across populations, it is quite likely that the item type is not measuring the same things for all populations.

Equal Standard Errors of Measurement Across Populations

The hypothesis here is that, in addition to having the same factors measured in the same units across populations, the diagonal elements of ψ are also equal across populations. The diagonal elements of ψ are estimates of the squared standard errors of measurement of each of the respective split-halved measures if, as has been shown, the split-halved scores are congeneric measures with equivalent scale units across populations. More formally, the hypothesis being tested is: $\psi_1 = \psi_2 =,\ldots,\psi_4$ conditional on equal factor loading patterns, scaling units, and intercepts.

This restricted model assumes that all elements of the factor model with the exception of the variance-covariance matrix among factors (ϕ) are equal across populations. These additional restrictions lead to an increment in χ^2 of 68 with 60 degrees of freedom ($p = .75$).

Table III below presents the root mean square residuals for this constrained model.

Table III

Root Mean Square Residuals by Population for the Factor Model That Assumes Equal Factor Loading Patterns, Intercepts, Units of Measurement, and Standard Error of Measurement Across Populations

	White Male	White Female	Black Male	Black Female
Root Mean Square Residuals for Means	.0591	.0604	.0330	.0359
Root Mean Square Residuals When the Variance-Covariance Matrix is Rescaled within Populations to be a Correlation Matrix	.0220	.0214	.0360	.0304

Average of All Root Mean Square Residuals = .0375

Clearly there is little additional "lack of fit" as measured by the increment in residual when the standard errors of measurement were constrained to be equal across populations. The 10 item types in the GRE Aptitude Test appear to be measuring all populations with equal precision.

The above three sequential tests of progressively "stronger" models, all of which provide a reasonably good fit, suggest that the GRE item types are measuring the same things in the same units with the same precision for all four populations. There does not appear to be any significant evidence of psychometric bias here. It should be remembered that psychometric bias as defined here is only one of many possible definitions of test bias. For other views see Darlington (1971) and Schmidt and Hunter (1976).

Equality of the Reliabilities of the Item-Type Factors

As pointed out earlier, the traditional estimates of internal consistency reliability are more a measure of the homogeneity of the populations than a measure of a particular instrument's accuracy. In the interest of completeness, however, the invariance across populations of the reliabilities of the 10 respective method factors was tested. This restricted model constrains corresponding main diagonal elements of the factor variance-covariance matrix (ϕ) to be equal across populations, in addition to the previous constraints of equality on the factor pattern matrices, intercepts, and main diagonal elements of ψ.

More formally, the hypothesis being tested is: $\phi_{iil} = \phi_{ii2} =, \ldots, \phi_{iig}$ conditional on equal factor loading patterns, scale units, intercepts, and standard errors of measurement. This model led to an increment in χ^2 of 67 with 30 degrees of freedom ($p \tilde{=} .001$) over the previous model.

Table IV presents the root mean square residuals.

Table IV

Root Mean Square Residuals by Population for the Factor Model That Assumes Equal Factor Loading Patterns, Intercepts, Units of Measurement, Standard Errors of Measurement, and Reliability Across Populations

	White Male	White Female	Black Male	Black Female
Root Mean Square Residuals for Mean	.0551	.0601	.0350	.0361
Root Mean Square Residuals when the Variance-Covariance Matrix is Rescaled within Population to be a Correlation Matrix	.0452	.0235	.0388	.0554

Average of all Root Mean Square Residuals = .0436

The additional constraint on equal reliabilities does lead to a slightly greater increment in fit over the previous model, yet the average residual suggests that the model is still quite reasonable. Although we feel that the residuals are sufficiently small to accept the equal reliabilities model, an inspection of the reliabilities of the various item types based on the previous analysis (unconstrained reliabilities) might be informative.

The reliabilities of the item types (factors) conditional on the factor model are computed as:

$$\hat{P}_k = [(\Sigma\Lambda_i)^2\phi_k] / [(\Sigma\Lambda_i)^2\phi_k + \Sigma\Psi_i] \qquad (5)$$

Table V presents the reliabilities by item type by population.

Table V

Reliabilities by Population for the Factor Model That Assumes Equal Factor Patterns, Intercepts, Units of Measurement, and Standard Errors of Measurement Across Populations

	Sentence Completion 1	Analogies 2	Antonyms 3	Reading 4	Quantitative Comparisons 5	Regular Math 6	Data Interpretation 7	Analysis of Explanations 8	Logical Diagrams 9	Analytical Reasoning 10
White Males	.7727	.7890	.8033	.8288	.7313	.6845	.7145	.8437	.7951	.7776
White Females	.7900	.7643	.7789	.8140	.6688	.6486	.6283	.8469	.8032	.7584
Black Males	.7672	.7288	.7840	.8073	.6997	.6424	.5364	.8510	.7895	.7074
Black Females	.7832	.6642	.7053	.8086	.6537	.5855	.5661	.8402	.8671	.7041
# of Items of Each Item Type	17	18	20	25	30	15	10	40	14	15

Inspection of Table V suggests little in the way of consistent patterns, although there is some tendency for scores of Blacks to to have somewhat lower reliabilities than the corresponding scores of Whites. Similarly, scores of White females and to somewhat lesser extent those of Black females tend to have slightly lower reliabilities than the scores of their male counterparts.

Equal Factor Model

This constrained model includes all the previous constraints and adds the constraint of equal factor covariances. Formally: $\Sigma_g = \Lambda\phi\Lambda' + \psi$. The increment in χ^2 is 201 with 135 degrees of freedom ($p \cong .001$). Table VI presents the root mean square residuals for this very constrained model.

Table VI

Root Mean Square Residuals by Population for the Factor Model That Assumes Same Factor Patterns, Intercepts, Equal Units of Measurement, Standard Errors of Measurement, True Variances, and Covariances Across Populations

	White Male	White Female	Black Male	Black Female
Root Mean Square Residuals for Means	.0553	.0598	.0351	.0361
Root Mean Square Residuals When The Variance-Covariance Matrix is Rescaled Within Population to be a Correlation Matrix	.0452	.0235	.0462	.0439

Average of All Root Mean Square Residuals = .0440

Inspection of the residuals in Table VI supports a reasonable fit for this model. In a certain sense this is a stronger model than one requiring equality of the observed variance-covariance matrices across populations since we are further imposing a specific factor structure dictated by the original test construction specifications. The factor loading patterns and intercorrelations among factors for this fully constrained model are presented in Appendices B and C.

Since the residuals show no practical differences from zero, we can use the pooled population estimate of Σ under the most constrained model to estimate the reliabilities and standard errors of measurement of each item-type factor. The previous reliability estimates were obtained from the less constrained model, which allowed the reliabilities to vary across populations. Table VII shows the reliabilities, standard errors of measurement, and number of items for each item type under this most constrained model.

Table VII

Reliabilities and Standard Errors of Measurement for the Factor Model That Assumes Same Factor Patterns, and Equal Intercepts, Units of Measurement, Standard Errors of Measure, True Variance, and Covariances Across Populations

	Sentence Completion	Analogies	Antonyms	Reading	Quantitative Comparisons	Regular Math	Data Interpretation	Analysis of Explanations	Logical Diagrams	Analytical Reasoning
Reliabilities	.78	.74	.77	.82	.69	.64	.62	.85	.79	.74
Standard Error of Measurement	1.60	1.51	1.45	1.99	2.32	1.45	1.11	2.59	1.44	1.48
No. of Items of Each Item Type	17	18	20	25	30	15	10	40	14	13

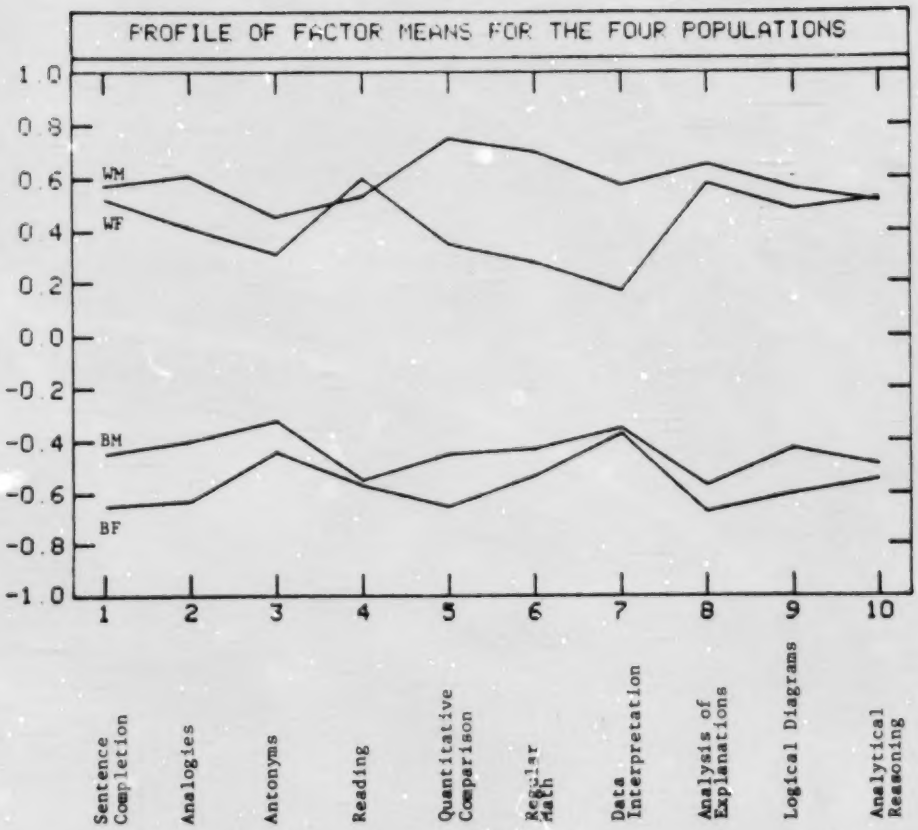
Factor Means

It is customary in item-group interaction studies to define items that are exceptionally hard for one or more subpopulations as biased in some sense. These items are then inspected to identify possible causes for their acting differently for a particular population. Since information on covariances between items is not taken into consideration in establishing evidence for whether or not the items are measuring the same things in the same scale units, the finding of differentially difficult items may or may not indicate bias. If it can be established through the analysis of the covariance structures that items, or logical subsets of items, appear to be measuring the same things in the same scale units, then the finding of differential difficulty more likely implies differential achievement rather than bias.

If one starts out with the assumption that the item types describe different possible ways for processing verbal, mathematical, and analytical information, and if the data are consistent with an invariant factor structure across populations, then, in general, the interpretation of differences in factor means as differential levels of achievement would appear to be reasonable.

With this in mind, Figure 3 presents profiles of factor means by population for the 10 item types. The factor scores are scaled in terms of standard deviation units with a grand mean of zero. Inspection of Figure 3 indicates that there are group main effect differences, as well as some evidence for interaction between group and item-type difficulty. It would appear that White females do somewhat less well in all the quantitative sections while Black females do comparatively

Figure 3


PROFILE OF FACTOR MEANS FOR THE FOUR POPULATIONS
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
WM
WF
BM
BF
1
2
3
4
5
6
7
8
9
10
Sentence Completion
Analogies
Antonyms
Reading
Quantitative Comparison
Regular Math
Data Interpretation
Analysis of Explanations
Logical Diagrams
Analytical Reasoning

less well on quantitative comparisons. It appears that both Black males and females have slightly greater difficulty with the analysis of explanations item type than with the remaining two analytical item types. It should be noted that the interactions are quite small compared to the overall main effect differences.

Higher-Order Factor Analysis

Since the preceeding confirmatory tests suggested an invariant factor structure across populations, the resulting MLH estimate of the population invariant variance-covariance matrix was used in fitting the following higher-order factors. A single factor solution was run to provide base line indices of "goodness of Fit" to compare with the subsequent more theoretically appropriate models. The single factor solution shown below:

SINGLE FACTOR SOLUTION

Sentence Completion	.922
Analogies	.904
Antonyms	.821
Reading	.810
Quantitative Comparison	.706
Regular Math	.653
Data Interpretation	.576
Analysis of Explanations	.770
Logical Diagrams	.640
Analytical Reasoning	.726

Root mean square residual = .114

has a χ^2 to degrees of freedom ratio of 21.4. One could hardly expect a single factor solution to fit very well when both verbal and quantitative items are present in the factor analysis.

Three-Factor Solution

Table VIII presents the results of a confirmatory factor analysis of the three-factor model that is assumed to underlie the three section scores. Inspection of Table VIII indicates a reasonable fit of the three-factor solution yielding a root mean square residual of .066. However, the correlation between the quantitative factor and the analytical factor is .918 with a standard error of .069, indicating that we are observing a large amount of shared variance. The correlation between factors is corrected for attenuation. That is, the .918 represents the correlation between quantitative ability and analytical ability as measured by their respective item types when both sets of measures are corrected for unreliability. Using equation (5), the estimated reliabilities of the factors are .95, .87, and .84 for the verbal, quantitative, and analytical factors respectively. Given reliabilities of this magnitude, one could expect the observed correlation between analytical and quantitative scores (i.e., correlations between quantitative and analytical scores not corrected for attentuation) to be in the high seventies. Since the maximum likelihood factor analysis model considers all unique variance under the constrained model to be error variance, the correlation corrected for attenuation between factors such as the quantitative factor and the analytical factor, whose indicators possess relatively large amounts of method (unique) variance, tends to be high.

Table VIII

Higher Order Confirmatory Factor Analysis of the Model Underlying the Three Section Scores

	V	Q	A
Sentence Completion	0.963	0.0	0.0
Analogies	0.950	0.0	0.0
Antonyms	0.863	0.0	0.0
Reading	0.766	0.0	0.0
Quantitative Comparison	0.0	0.865	0.0
Regular Math	0.0	0.880	0.0
Data Interpretation	0.0	0.733	0.0
Analysis of Explanations	0.0	0.0	0.824
Logical Diagrams	0.0	0.0	0.735
Analytical Reasoning	0.0	0.0	0.840

Intercorrelations Among Factors

	V	Q	A
V	1.000		
Q	0.636	1.000	
A	0.769	0.918	1.000

Root Mean Square Residual = .066

Conversely, internal consistency estimates of reliability (especially split-halves methods) consider shared variance due to the same method being present in both halves (e.g., split halves of the same item type) as true variance. These lower bound estimates of reliability could just as appropriately be referred to as indices of the construct validity of the weighted composites defining each factor. When congeneric (i.e., different methods of measuring the same construct) rather than parallel measures are constrained to define factors, the difference between reliability and construct validity becomes a "grey" area if not a meaningless differentiation. We will, however, continue to refer to such indices as reliability to be consistent with Jöreskog's (1971) original factorial-based definition of reliability. The important point here is that when a factor model is constrained to isolate method variance as error variance, the correlation between factors is likely to be quite high. The fact that it is so high in this case is not surprising since one measure of the quantitative factor (data interpretation) and one measure of the analytical factor (logical diagrams) have relatively large unique components of method variance.

Given the considerations above, the correlation between the verbal and quantitative factors is comparatively low. This particular pattern of between-factor correlations might be due in part to the selected population we are dealing with here. That is, if one used the unselected GRE population (i.e., not just social science majors), one might observe a somewhat lower correlation between the quantitative and analytical factors and, conversely, a higher correlation between verbal and quantitative.

Although this three-factor model fits reasonably well, there was a pattern of residuals associated with the reading comprehension items that could not be considered zero. That is, after controlling for the verbal component in reading comprehension (as defined by the first factor), there remained correlations (as indicated in the matrix of residuals) between reading and measures of both quantitative ability and analytic ability. These correlated residuals run from a low of .10 with regular mathematics to a high of .20 with analysis of explanations. The χ^2 to degree of freedom ratio here was slightly over 8.

In an effort to yield a better "fit", (i.e., reduce residual correlations between reading, mathematical, and analytical items as well as further investigate the relationship between reading and the other factors), a four-factor confirmatory model was hypothesized, with reading being a separate factor.

Table IX presents the four-factor solution, which indeed does reduce the root mean square residuals from .060 to .043 and, not surprisingly, reduces all the reading related residuals to essentially zero. The χ^2 to degrees of freedom ratio is approximately 7.

Table IX

Higher Order Confirmatory Factor Analysis with Reading as a Separate Factor

		V	R	Q	A
X_1	Sentence Completion	0.955	0.0	0.0	0.0
X_2	Analogies	0.961	0.0	0.0	0.0
X_3	Antonyms	0.873	0.0	0.0	0.0
X_4	Reading	0.0	1.000	0.0	0.0
X_5	Quantitative Comparison	0.0	0.0	0.864	0.0
X_6	Regular Math	0.0	0.0	0.880	0.0
X_7	Data Interpretation	0.0	0.0	0.733	0.0
X_8	Analysis of Explanation	0.0	0.0	0.0	0.825
X_9	Logical Diagrams	0.0	0.0	0.0	0.733
X_{10}	Analytical Reasoning	0.0	0.0	0.0	0.840

Intercorrelations Among Factors

	V	R	Q	A
V	1.000			
R	0.744	1.000		
Q	0.623	0.645	1.000	
A	0.748	0.793	0.918	1.000

Root Mean Square Residual = .043

Inspection of intercorrelations among the factors in Table IX indicates that reading has a higher relationship with the analytical factor than it does with the verbal factor. It could well be that the analytical factor is itself a complex construct sharing variance with reading comprehension (certainly understanding the directions for analysis of explanations requires a heavy reading load) and the abstract reasoning present in some mathematical items such as the quantitative comparison items.

Although the above four-factor solution provides information on the relationship between the analytical factor and the other factorial components of the GRE Aptitude Test, it does not provide much information on the relationship of the separate analytical item types and the other traditional factors. Table X presents a three-factor confirmatory solution where, as before, the first factor was defined by the three verbal indicators, a second factor was defined by the reading items, and the third factor was the quantitative factor defined by the three quantitative item types. The three analytical item types were left free to load according to the maximum likelihood criterion of best fit. Inspection of Table X indicates that much of the shared common variance among the three analytical item types is also common to the quantitative item types. This, of course, is consistant with the finding of the high unattenuated correlation between the quantitative factor and the analytical factor from the four-factor solution.

The analysis of explanations item type appears to be somewhat more factorially complex than either logical diagrams or analytical reasoning. That is, it has small loadings on both the verbal and reading factors as well as a substantial loading on the quantitative factor. Analytical reasoning and logical diagrams have essentially zero loadings on the verbal factor, small

Table X

Higher Order Confirmatory Factor Analysis
with the Analytical Items Free to Load on
the Verbal, Reading, and Quantitative Factors

	V	R	Q
Sentence Completion	0.956	0.0	0.0
Analogies	0.959	0.0	0.0
Antonyms	0.872	0.0	0.0
Reading	0.0	1.000	0.0
Quantitative Comparison	0.0	0.0	0.869
Regular Math	0.0	0.0	0.861
Data Interpretation	0.0	0.0	0.723
Analysis of Explanation	0.199	0.230	0.470
Logical Diagrams	0.041	0.175	0.555
Analytical Reasoning	0.046	0.155	0.697

Intercorrelations Among Factors

	V	R	Q
V	1.000		
R	0.746	1.000	
Q	0.631	0.654	1.000

Root Mean Square Residual = .044

loadings on the reading factor and substantial loadings on the quantitative factor. It is interesting that, among the analytical items, analytical reasoning seems to share the most variance with the quantitative items. These results suggest that the variance shared by the three analytical measures is relatively highly correlated with the quantitative factor and also correlated to a lesser extent with the reading factor.

Inspection of the uniquenesses (Ψ_i) suggests that data interpretation (Ψ=.48), analysis of explanations (Ψ=.36), and logical diagrams (Ψ=.49) have comparatively large unique components. When a separate analytical factor is defined (as in the four-factor solution), the unique variance in analysis of explanations was somewhat reduced (Ψ=.32), but the unique variance in data interpretation (Ψ=.46) and logical diagrams (Ψ=.46) remained relatively high. It would appear that both data interpretation and logical diagrams have components that are measuring something not covered by the other item types. It is unlikely that this unique variance is entirely error variance since the reliabilities (see Table VII) are comparatively high for the number of items in these scales. At this point it would appear that data interpretation, logical diagrams, and, to a lesser extent, analysis of explanations have comparatively larger unique components of method variance than the remaining item types.

Table XI presents the extensions of the four-factor solution to three background variables from the background information questionnaire (Altman, 1977). The extension procedure was carried out separately for each of the four subpopulations. The three background variables were graduate degree aspirations (item J), self-reported grades in undergraduate major (item O), and self-reported overall grades for the junior and senior years (item P).

Table XI

Factor Extensions on Academic Plans and Undergraduate Grades

Biographical Items	Populations			
	White Males			
	Verbal	Reading	Quantitative	Analytical
Item J	.148	.166	.100	.149
" O	.224	.277	.210	.276
" P	.217	.301	.217	.285
	White Females			
Item J	.198	.205	.152	.195
" O	.250	.293	.194	.284
" P	.234	.263	.189	.267
	Black Males			
Item J	.184	.117	.118	.140
" O	.083	.052	.061	.116
" P	.189	.135	.145	.215
	Black Females			
Item J	.178	.173	.151	.167
" O	.064	.081	.078	.071
" P	.154	.153	.127	.177

The question here is whether the same patterns of relationship hold between the extension variables and the four factors within all subpopulations. Inspection of Table XI indicates that, with one interesting exception, the pattern of correlations is consistent across populations. The exception is that the four factors have a proportionately lower relationship with grades in undergraduate major than with overall junior and senior grades for the two Black sex groups. It is possible that, although this analysis has been restricted to social science majors, some variation still may remain between subpopulations in the major fields being emphasized. If this is true, then overall junior and senior grades may be more comparable across populations.

There is a slight tendency for females to show a higher relationship between their factor scores and their long-term academic aspirations (Item J). This may reflect in part a greater propensity of males to pursue a particular career for reasons other than that they seem to have the tested abilities. In general, the pattern of correlations between the demographics and verbal, quantitative, and analytical ability scores are similar but lower than those found by Miller and Wild (1979) for social science majors.

It is somewhat encouraging to note that, while the analytical factor showed a high unattenuated correlation with the quantitative factor, it showed a higher "validity" with overall junior and senior grades for all subpopulations than did the quantitative factor. In fact, the analytical factor showed a higher relationship with past academic performance (grades in junior and senior year) than did any of the other factors with the exception of reading.

The results of the extension are consistent with the previous internal analysis, in that the pattern of the extension coefficients is relatively

invariant across populations (with the one noted exception). The fact that the analytical factor shows a slightly higher "validity" than other factors, except for reading, with past academic performance is not surprising since the shared variance in the analytical factor is relatively complex, being related to both the quantitative and reading skill factors. Complex constructs (such as the analytical factor), as opposed to single-factor measures, are likely to have higher relationships with complex criteria.

The results of this study are for the most part consistent with the results of the Swinton and Powers (1980) factor analytic study of the restructured GRE Aptitude Test. Using exploratory factor analytic procedures, Swinton and Powers defined the following oblique factors: (1) reading comprehension, (2) vocabulary, (3) quantitative ability, and (4) analytical reasoning. Similar to the results of the present study, the analytical factor was found to be highly correlated with the reading and quantitative factors. Although the absolute magnitude of the correlations was lower than that found in the present confirmatory study, this discrepancy can be partially explained by differences in methodological approaches. Swinton and Powers found that the analysis of explanations item type was internally complex while our confirmatory procedures also suggested that, among the analytical item types, it was the most complex. The finding by both studies of a separate reading factor relatively highly correlated with the analytical factor (and, to a lesser extent, with the other factors) yielded independent evidence of the primacy of the reading construct in all test items. Swinton and Powers found that past academic achievement was positively related to all GRE Aptitude Test factors, as did this study.

With respect to logical diagrams, Swinton and Powers found them to be factorially complex, although apparently less so than analysis of explanations. This study also found them to be somewhat complex (and less so than analysis of explanations) but to also have a relatively large component of unique (possibly method) variance.

Comparison of ethnic and sex differences from the two studies showed similar results. Although not specifically shown, Swinton and Powers state that the ranks of the ethnic groups remain relatively stable across their factors. Similar results are suggested from the factor scores of this study. The Blacks' relative position on the factors is fairly stable, with a minor drop on the quantitative and analysis of explanations items. Sex group factor score means lead to similar conclusions for both studies, with the exception that, in this study, White female means do not exceed White male means except for reading comprehension. These slight differences in findings are possibly due to the confounding of sex and ethnic differences with fields of study.

In summary, the two studies arrive at reasonably similar conclusions using quite dissimilar methodologies and samples. The Swinton and Powers study examined the factorial structure of the GRE Aptitude Test within a single heterogeneous population. They then investigated a number of relationships between external biographical information and the obtained factor structure within that population. The present study investigated the possibility of an invariant GRE factor structure across sex and ethnic groups, controlling for major field of study. After developing a relatively invariant factor structure, a limited number of external biographical

variables were then related to the factor structure within each population. The results of the confirmatory study developed additional evidence for the presence and complexity of the factors identified in the Swinton and Powers study and further demonstrated the invariance of selected psychometric characteristics of the factors across ethnic and sex groups.

Summary and Conclusions

The primary purpose of this study was to: (1) evaluate the invariance of the internal structure construct validity and thus the interpretation of GRE Aptitude Test scores across four populations, and (2) develop and apply a systematic procedure for investigating the possibility of test bias from a construct validity frame of reference. The notion of invariant construct validity was defined as (1) similar patterns of loadings across populations, (2) equal units of measurement across populations, and (3) equal test score precision as defined by the standard error of measurement. Although other forms of bias might exist that would not be identified by these procedures, if any one of the above criteria differs across populations, then one has to consider seriously the possibility of psychometric bias, as defined in this paper. The advantage of investigating psychometric bias at the item type level (even though the total score may not be biased) is that this may provide an "early warning" with respect to any future plans to increase the number of items of any particular type. A secondary purpose of this study was to evaluate the factor structure of the three sections (verbal, quantitative and analytical) from which section scores are derived. Assuming

that the invariant construct validity model based on item types is tenable, a hypothesized three factor "macro" model based on the three sections could be carried out on the population invariant variance-covariance matrix.

It should be noted that the term "psychometric bias" as defined here does not require external criteria information for the analysis. The internal procedure used here is suggested as only a first step in a broader process of an integrated validation procedure that should include not only internal checks on the population invariance of the underlying constructs but also checks on the population invariance of their relationships with external criteria. Although this is only a first step, it is a necessary step since any interpretation of relationships with external criteria becomes academic unless one can first show that the tests measure what they purport to measure with similar meaning and accuracy for all populations of interest.

The four subpopulations were 1,122 White males, 1,471 White females, 284 Black males, and 626 Black females.

The analysis indicated that a factor structure defined by the 10 item types showed relatively invariant psychometric characteristics across the four subpopulations. That is, the item-type factors appear to be measuring the same things in the same units with the same precision. There does not appear to be any significant evidence of psychometric bias in the test.

Confirmatory analysis of a higher-order factor model defined by an a priori model based on three- and four-factor solutions was attempted to investigate the factorial contributions of the analytical item types.

Results of this analysis indicated that the three analytical item types appear to be varying functions of reading comprehension and quantitative ability. The analysis of explanations item type was the more complex factorially and included a vocabulary component as well as reading and quantitative components. Of the remaining two analytic item types, logical diagrams had the comparatively larger unique variance component. Analytical reasoning appear to share most of its variance with the reading comprehension and quantitative factors.

It would seem that of the analytical item types, logical diagrams has the greatest possibility of adding unique yet reliable variance to the GRE Aptitude Test while analytical reasoning items appear to add the least amount of new information. Analysis of explanations is the most factorially complex but its multidimensionality is to a great extent described by the already present verbal, reading comprehension, and quantitative factors. For other views see Darlington (1971) and Schmidt and Hunter (1976).

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Appendix A

Examples of the 10 Item Types Used in the GRE Aptitude Test

Verbal Ability

ANALOGIES

Questions of this type test the ability to understand relationships among words and ideas.

Directions: In each of the following questions, a related pair of words or phrases is follwed by five lettered pairs of words or phrases. Select the lettered pair which best expresses a relationship similar to that expressed in the original pair.

Example:

COLOR:SPECTRUM:: (A) tone:scale (B) sound:waves
(C) verse:poem (D) dimension:space (E) cell:organism

ANTONYMS

Questions of this type test the extent of the student's vocabulary.

Directions: Each question below consists of a word printed in capital letters followed by five words or phrases lettered A through E. Choose the lettered word or phrase that is most nearly opposite in meaning to the word in capital letters. Since some of the questions require you to distinguish fine shades of meaning, be sure to consider all the choices before deciding which one is best.

Example:

PROMULGATE: (A) distort (B) demote (C) suppress
(D) retard (E) discourage

SENTENCE COMPLETION

This type of question provides a measure of one aspect of reading comprehension: the ability to recognize logical and stylistic consistency among the elements in a sentence.

Directions: Each of the sentences below has one or more blank spaces, each blank indicating that a word has been omitted. Beneath the sentence are five lettered words or sets of words. You are to choose the one word or set of words which, when inserted in the sentence, best fits in with the meaning of the sentence as a whole.

Example:

Early ------- of hearing loss is ------- by the fact that the other senses are able to compensate for moderate amounts of loss, so that people frequently do not know that their hearing is imperfect.

(A) discovery..indicated (B) development..prevented
(C) detection..complicated (D) treatment..facilitated
(E) incidence..corrected

READING COMPREHENSION

Reading passages are taken from a variety of fields, and reading comprehension is tested at several levels. Some of the questions merely test understanding of the plain sense of what has been stated. Others ask for interpretation, analysis, or application of the principles or opinions expressed by the author. The reading passages may be either shorter or longer than the sample passage presented below.

Directions: Each passage is followed by questions based on its content. After reading the passage, choose the best answer to each question. Answer all questions following a passage on the basis of what is stated or implied in that passage.

Example:

In the years following the Civil War, economic exploitation for the first time was provided with adequate resources and a competent technique, and busy prospectors were daily uncovering new sources of wealth. The coal and oil of Pennsylvania and Ohio, the copper and iron ore of Upper Michigan, the gold and silver, and the lumber and fisheries of the Pacific Coast provided limitless raw materials for the rising industrialism. The Bessemer process quickly turned an age of iron into an age of steel and created the great mills of Pittsburgh from which issued the rails for expanding railways. The reaper and binder, the sulky plow, and the threshing machine created a large scale agriculture on the fertile prairies. Wild grasslands provided grazing for immense herds of cattle and sheep; the development of the corn belt enormously increased the supply of hogs; and with railways at hand the Middle Border poured into Omaha and Kansas City and Chicago an endless stream of produce.

As the line of the frontier pushed westward, new towns were built, thousands of claims to homesteads were filed, and speculator and promoter hovered over the prairies like buzzards seeking their carrion. With rising land values money was to be made out of unearned increment, and the creation of booms was a profitable industry. The times were stirring, and it was a shiftless fellow who did not make his pile. If he had been too late to file on desirable acres, he had only to find a careless homesteader who had failed in some legal technicality and "jump his claim." Good bottom land could be had even by late-comers if they were sharp at the game.

The bustling America of 1870 accounted itself a democratic world. A free people had put away all aristocratic privileges and, conscious of power, had gone forth to possess the last frontier. But America's essential social philosophy, which it found adequate to its needs, was summed up in three words -- preemption, exploitation, progress. Its immediate and pressing business was to dispossess the government of its rich holdings. Lands in the possession of the government were so much idle waste, untaxed and profitless; in private hands they would be developed. They would provide work, pay taxes, support schools, enrich the community. Preemption meant exploitation and exploitation meant progress.

It was a simple philosophy and it suited the simple individualism of the times. The Gilded Age knew nothing of enlightenment; it recognized only the acquisitive instinct. That much at least the frontier had taught the great American democracy; and in applying to the resources of a continent the lesson it had been so well taught, the Gilded Age wrote a profoundly characteristic chapter of American history.

According to the passage, increased corn production was mainly responsible for an increase in the

(A) number of sheep (B) output of farm implements
(C) supply of hogs (D) amount of pasture land
(E) number of cattle

Quantitative Ability

REGULAR MATH AND GRAPHS

Directions: Solve each of the following problems, using any available space on the page for scratch work. Then indicate the best answer in the appropriate space on the answer sheet.

Note: Figures which accompany these problems are intended to provide information useful in solving the problems. They are drawn as accurately as possible EXCEPT when it is stated in a specific problem that the figure is not drawn to scale. All figures lie in a plane unless otherwise indicated.

All numbers are real numbers.

Example 1: Regular Math

The average of x and y is 20. If z = 5, what is the average of x, y, and z?

(A) 8 1/3 (B) 10 (C) 12½ (D) 15 (E) 17½

Example 2: Graphs

PER CENT CHANGE IN DOLLAR AMOUNT OF SALES IN CERTAIN RETAIL STORES FROM 1977 TO 1979

Store	Per Cent Change	
	From 1977 to 1978	From 1978 to 1979
P	+10	-10
Q	-20	+ 9
R	+ 5	+12
S	- 7	-15
T	+17	- 8

In 1979 which of the stores had greater sales than any of the others shown?

(A) P (B) Q (C) R (D) S
(E) It cannot be determined from the information given

QUANTITATIVE COMPARISONS

Directions: Each question in this part consists of two quantities, one in Column A and one in Column B. You are to compare the two quantities and on the answer sheet blacken space

A if the quantity in Column A is the greater;

B if the quantity in Column B is the greater;

C if the two quantities are equal;

D if the relationship cannot be determined from the information given.

Common Information: In a question, information concerning one or both of the quantities to be compared is centered above the two columns. A symbol that appears in both columns represents the same thing in Column A as it does in Column B.

Numbers: All numbers used are real numbers; all square roots are positive numbers.

Figures: Position of points, angles, regions, etc. can be assumed to be in the order shown.

Lines shown as straight can be assumed to be straight.

Figures are assumed to lie in the plane unless otherwise indicated.

Figures which accompany questions are intended to provide information useful in answering the questions. However, unless a note states that a figure is drawn to scale, you should solve these problems NOT by estimating sizes by sight or by measurement, but by using your knowledge of mathematics.

Example:

Column A	Column B
2 x 6	2 + 6

Analytical Ability

ANALYSIS OF EXPLANATIONS

Directions: For each set of questions, a fact situation and a result are presented. Several numbered statements follow the result. Each statement is to be evaluated in relation to the fact situation and result.

Consider each statement separately from the other statements. For each one, examine the following sequence of decisions, in the order A,B,C,D,E. Each decision results in selecting or eliminating a choice. The first choice that cannot be eliminated is the correct answer.

A Is the statement inconsistent with, or contradictory to, something in the fact situation, the result, or both together? If so, choose A.
If not,

B Does the statement present a possible adequate explanation of the result? If so, choose B.
If not,

C Does the statement have to be true if the fact situation and result are as stated? If so, the statement is deducible from something in the fact situation, the result, or both together; choose C.
If not,

D Does the statement either support or weaken a possible explanation of the result? If so, the statement is relevant to an explanation: choose D.

E If not, the statement is irrelevant to an explanation of the result; choose E.

Use common sense to decide whether explanations are adequate and whether statements are inconsistent or deducible. No formal system of logic is presupposed. Do not consider extremely unlikely or remote possibilities.

Example:

Situation. In an attempt to end the theft of books from Parkman University Library, Elnora Johnson, the chief librarian, initiated a stringent inspection program at the beginning of the fall term. At the library entrance, Johnson posted inspectors to check that each library book leaving the building had a checkout slip bearing the call number of the book, its due date, and the borrower's identification number. The library retained a carbon copy of this slip as its only record that the book had been checked out. Johnson ordered the inspectors to search for concealed library books in attaché cases, bookbags, and all other containers large enough to hold a book. Since no new personnel could be hired, all library personnel took turns serving as inspectors, though many complained of their embarrassment in conducting the searches.

Result. During that term Margaret Zimmer stole twenty-five library books.

Statement. Zimmer stole the books before the inspection began.

LOGICAL (VENN) DIAGRAMS

Directions: In this part, you are to choose from five diagrams the one that illustrates the relationship among three given classes better than any of the other diagrams offered.

There are three possible relationships between any two different classes:

indicates that one class is completely contained in the other but not vice versa.

indicates that neither class is completely contained in the other, but the two have members in common.

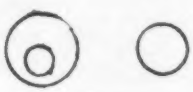
indicates that there are no members in common.

Note: The size of the circles does not indicate relative size of the classes.

Example:

Birds, robins, trees

(A)



(B)

(C)

(D)

(E)

ANALYTICAL REASONING

Directions: Each question or group of questions is based on a passage or set of statements. In answering some of the questions it may be useful to draw a rough diagram. Choose the best answer for each question and blacken the corresponding space on your answer sheet.

Example:

(1) It is assumed that a half tone is the smallest possible interval between notes.

(2) Note T is a half tone higher than note V.

(3) Note V is a whole tone higher than note W.

(4) Note W is a half tone lower than note X.

(5) Note X is a whole tone lower than note T.

(6) Note Y is a whole tone lower than note W.

Which of the following represents the relative order of the notes from lowest to highest?

(A) X Y W V T (B) Y W X V T (C) W V T Y X
(D) Y W V T X (E) Y X W V T

Appendix B

Factor Loading Pattern for the Fully Constrained Model

		Sentence Completion	Analogies	Antonyms	Reading	Quantitative Comparison	Regular Math	Data Interpretation	Analysis of Explanations	Logical Diagrams	Analytical Reasoning
Sentence Completion	O	1.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	E	0.80	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Analogies	O	0.0	1.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	E	0.0	1.07	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Antonyms	O	0.0	0.0	1.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	E	0.0	0.0	0.83	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Reading	O	0.0	0.0	0.0	1.00	0.0	0.0	0.0	0.0	0.0	0.0
	E	0.0	0.0	0.0	1.02	0.0	0.0	0.0	0.0	0.0	0.0
Quantitative Comparison	O	0.0	0.0	0.0	0.0	1.00	0.0	0.0	0.0	0.0	0.0
	E	0.0	0.0	0.0	0.0	0.78	0.0	0.0	0.0	0.0	0.0
Regular Math	O	0.0	0.0	0.0	0.0	0.0	1.00	0.0	0.0	0.0	0.0
	E	0.0	0.0	0.0	0.0	0.0	0.92	0.0	0.0	0.0	0.0
Data Interpretation	O	0.0	0.0	0.0	0.0	0.0	0.0	1.00	0.0	0.0	0.0
	E	0.0	0.0	0.0	0.0	0.0	0.0	1.10	0.0	0.0	0.0
Analysis of Explantion	O	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.00	0.0	0.0
	E	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.91	0.0	0.0
Logical Explanations	O	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.00	0.0
	E	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.00	0.0
Analytical Reasoning	O	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.00
	E	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.22

Appendix C

Intercorrelations among Factors for the Fully Constrained Factor Model

Sentence Completion	1.000									
Analogies	0.908	1.000								
Antonyms	0.816	0.862	.000							
Reading	0.766	0.671	0.586	1.000						
Quantitative Comparison	0.552	0.538	0.480	0.588	1.000					
Regular Math	0.490	0.503	0.464	0.538	0.768	1.000				
Data Interpretation	0.449	0.450	0.374	0.493	0.569	0.733	1.000			
Analysis of Explanations	0.676	0.613	0.531	0.683	0.662	0.583	0.506	1.000		
Logical Diagrams	0.543	0.481	0.428	0.577	0.670	0.542	0.434	0.680	1.000	
Analytical Reasoning	0.580	0.579	0.465	0.643	0.711	0.692	0.639	0.665	0.598	1.000

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